Section 3-4
The Slope of a Line: Parallel and Perpendicular Line

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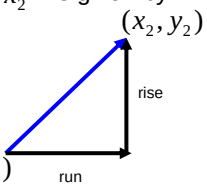
Objectives

- Find the slope of a line given two points
- Find the slope of a line given the equation of the line
- Determine whether two lines are parallel, perpendicular, or neither
- Solve applications involving slope

Finding Slopes from Two Points

- The ***slope m*** of the line going through the points (x_1,y_1) and (x_2, y_2) where $x_1 \neq x_2$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$



- The ***slope m*** of the line is the change of y over the change of x.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples of Slope (Steepness)






Finding the Slope of Two Points

$(-6,0)\quad (x_1, y_1) = (-6,0)$

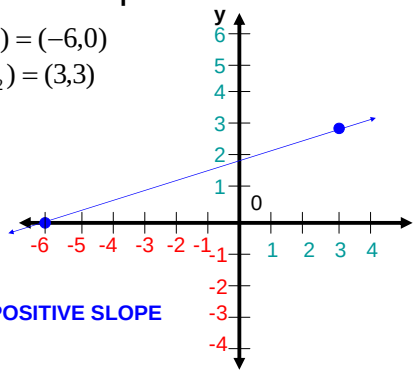
$(3,3)\quad (x_2, y_2) = (3,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-0}{3-(-6)}$$

$$m = \frac{3}{9} = \frac{1}{3}$$

POSITIVE SLOPE



Finding the Slope of Two Points

$(-2,3)\quad (x_1, y_1) = (-2,3)$

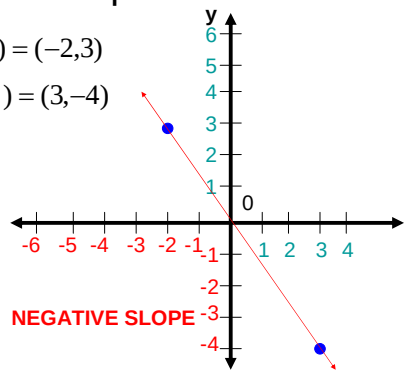
$(3,-4)\quad (x_2, y_2) = (3,-4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4-3}{3-(-2)}$$

$$m = \frac{-7}{5} = -\frac{7}{5}$$

NEGATIVE SLOPE



Finding the Slope of Two Points

$(-2,-2)\quad (x_1, y_1) = (-2,-2)$

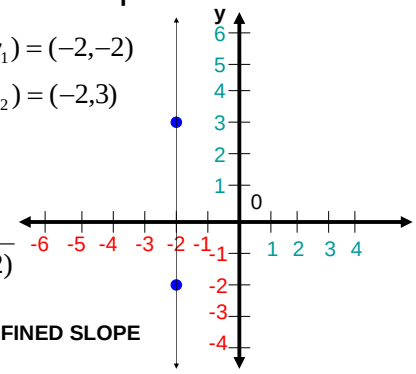
$(-2,3)\quad (x_2, y_2) = (-2,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-(-2)}{(-2)-(-2)}$$

$$m = \frac{5}{0}$$

UNDEFINED SLOPE



Finding the Slope of Two Points

$(1,4)\quad (x_1, y_1) = (1,4)$

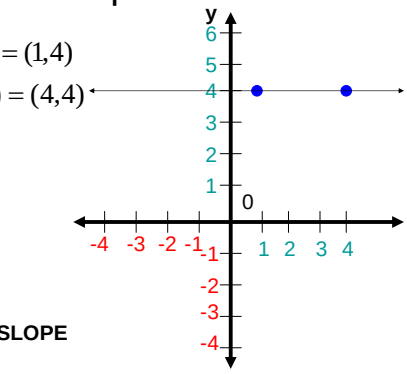
$(4,4)\quad (x_2, y_2) = (4,4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-4}{4-1}$$

$$m = \frac{0}{3}$$

ZERO SLOPE



Finding Slopes of *y=mx+b*

- The ***slope m*** of the line defined by $y = mx + b$ is *m*.
- Finding a slope.
- 1. Solve the equation for *y*
- 2. The slope is *m*, the coefficient of *x*.

Finding Slope *m* From Equations

$$2x + 3y = 6$$
$$2x + 3y - 2x = 6 - 2x$$
$$3y = 6 - 2x$$
$$\frac{3y}{3} = \frac{6 - 2x}{3}$$
$$y = 2 + \left(-\frac{2}{3}\right)x \qquad y = -\frac{2}{3}x + 2 \qquad m = -\frac{2}{3}$$
$$y = mx + b$$

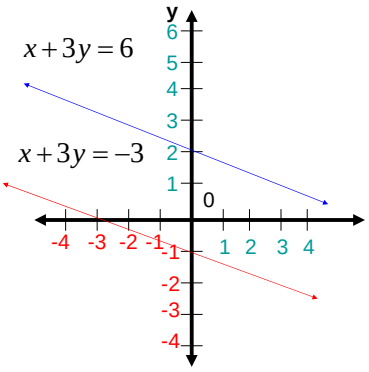
Finding Slope *m* From Equations

$$3x - 2y = 4$$
$$3x - 2y - 3x = 4 - 3x$$
$$-2y = 4 - 3x$$
$$\frac{-2y}{-2} = \frac{4 - 3x}{-2}$$
$$y = -2 + \frac{3}{2}x \qquad y = \frac{3}{2}x - 2 \qquad m = \frac{3}{2}$$
$$y = mx + b$$

Parallel Lines

- Two lines with the same slope but different y-intercepts are parallel

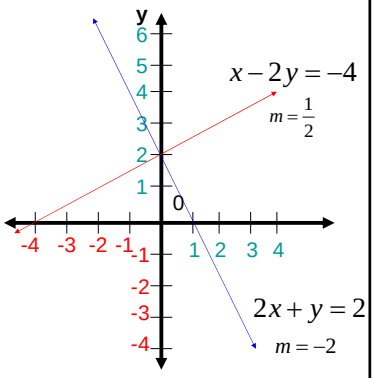
$$x + 3y = 6$$
$$x + 3y = -3$$
$$m = -\frac{1}{3}$$



Perpendicular Lines

- Two lines whose slopes have a product of -1 are perpendicular

$$x-2y=-4$$
$$x-2y-x=-4-x$$
$$-2y=-4-x$$
$$\frac{-2y}{-2}=\frac{-4-x}{-2}$$
$$y=\frac{1}{2}x+2$$

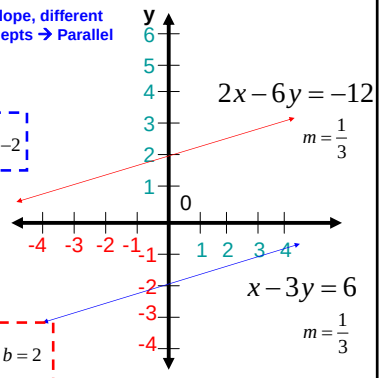


Parallel, Perpendicular Lines or Neither?

$$x-3y=6$$
$$x-3y-x=6-x$$
$$\frac{-3y}{-3}=\frac{6-x}{-3}$$
$$y=\frac{1}{3}x-2 \rightarrow m=\frac{1}{3} \quad b=-2$$

Same slope, different y-intercepts → Parallel lines

$$2x-6y=-12$$
$$2x-6y-2x=-12-2x$$
$$-6y=-12-2x$$
$$\frac{-6y}{-6}=\frac{-12-2x}{-6}$$
$$y=\frac{1}{3}x+2 \rightarrow m=\frac{1}{3} \quad b=2$$

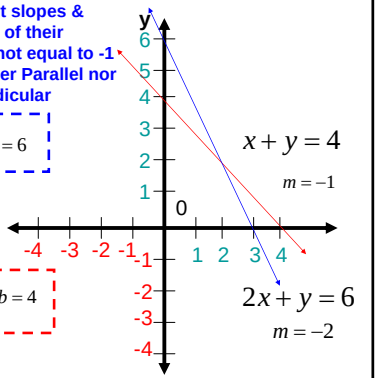


Parallel, Perpendicular Lines or Neither?

$$2x+y=6$$
$$2x+y-2x=6-2x$$
$$y=6-2x$$
$$y=-2x+6 \rightarrow m=-2 \quad b=6$$

Different slopes & product of their slopes not equal to -1 → Neither Parallel nor Perpendicular

$$x+y=4$$
$$x+y-x=4-x$$
$$y=4-x$$
$$y=-x+4 \rightarrow m=-1 \quad b=4$$

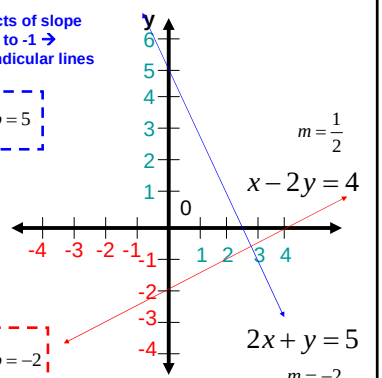


Parallel, Perpendicular Lines or Neither?

$$2x+y=5$$
$$2x+y-2x=5-2x$$
$$y=5-2x$$
$$y=-2x+5 \rightarrow m=-2 \quad b=5$$

Products of slope equals to -1 → Perpendicular lines

$$x-2y=4$$
$$x-2y-x=4-x$$
$$-2y=4-x$$
$$\frac{-2y}{-2}=\frac{4-x}{-2}$$
$$y=\frac{1}{2}x-2 \rightarrow m=\frac{1}{2} \quad b=-2$$



Applications Involving Slope

Escalating health care costs govern by the following equation

$$C = 17t + 127$$

where t is the number of years after 1980

a. What is the slope of C? $m = 17$

b. What does the slope represent?

The slope represents the annual increase of 17 dollars per year of daily room charge.